

APM466

Question 1. Consider a call option on a stock. The stock value today is S , has volatility σ , drift μ and the arbitrage-free option value when the stock is worth S at time t is $f(S, t, \mu, \sigma, r)$, where r is the risk-free constant interest rate, and the strike price for the option is K . Determine the sign of each of the following quantities. Justify your answer.

- $\partial_S^2 f$.
- $\partial_S f$.
- $\partial_\sigma f$.
- $\partial_\mu f$.
- $\partial_r f$.

Question 2. Andy likes to eat oranges, and Betty likes to eat apples. They shop at different markets, where the fruit prices are

	Apples	Oranges
Andy's market	10	15
Betty's market	11	13

Explain how Andy and Betty could use swaps to satisfy their fruit hungers at minimum cost. What would be fair prices for the exchange?

Question 3. Assume a certain stock is valued at \$1 today, and over a period of one year it will be worth either \$2 or \$0.5. Consider also a convertible bond on the stock, B_C , which has a value in one year equal to

$$\max(1, S_T).$$

- Determine which of the following three portfolios has lower VaR: B_C , $B_C + S$ or $B_C - S$?
- Determine the range of interest rates so that there are no arbitrage opportunities.

Question 4. The Wall Street Journal on July 1, 1999, listed the following values for the Treasury yield curve:

Maturity	Rate (%)
1 yr	5.07
2 yr	5.52
5 yr	5.66
10 yr	5.78
30 yr	5.97

You wish to sign a contract right now (July 1, 1999), under which you will pay $\$X$ one year from now, and be paid $\$100$ ten years from now. You do not want to pay or receive any money right now. What should the value of X be?

Question 5. Consider a portfolio with a counterparty with default probability of 12%. The portfolio has a stochastic exposure (in a year's time) given by a random variable with mean $\$100M$ and standard deviation of $\$10M$. The recovery rate is also random, with mean 50% and standard deviation given of 30%. If the recovery and exposure have a correlation of -70%, calculate the expected loss for the portfolio.