Stochastic Correlation in Energy Markets

Cui Cui Amanda Luo (University of Toronto), Haofei Wang (Reykjavik University), Desheng Wu (University of Stockholm), Luis Seco.

CAIMS, June 2016
Outline

1. Defaultable Weather Derivatives

2. Volatilities
   - Oil Markets

3. Stochastic Correlation
   - Stocks and Bonds
A snow swap

City

Snow

$10M

No snow

Resort
A swap portfolio

Consider a two-legged swap:

- A swap with the City, were $10M are exchanged as a function of snow precipitation in the City.
- A swap with the ski resort, where $10M are exchanged as a function of snow precipitation at the resort.

We charge 10% of the flow ($2M) as risk premium. We assume a 50% correlation between snow fall in the two locations.
Is this a good investment?

Our cashflows are:

- We post $20M as collateral, for $2M as a fee (10% rate of return)
- Standard deviation of 50%:
  - With 75% probability, the two swaps cancel each other’s cash flows
  - With 12.5% probability, we receive $10M from both the city and the resort
  - With 12.5% probability, we have to pay $10M to both the city and the resort
The snow fund

We perform 100 similar swaps,

- 10% expected return
- 5% standard deviation

1. Blue Mountain (Toronto)
2. Mountain Creek (New Jersey)
3. Panorama Mountain Village (Calgary)
4. Snowshoe Mountain (West Virginia)
5. Steamboat Ski Resort (Hayden, Denver)
6. Stratton Mountain Resort (Vermont)
7. Tremblant (Montreal)
8. Whistler Blackcomb (Vancouver)
9. etc.
Correlation breakdown

In 2008, Intrawest filed for bankruptcy

- 26 of our swap counterparties therefore are in default
- Assume a cold, snowy winter: $260M of lost swap revenue
- From $2Bn invested, this is a -3% RoR.
Stochastic volatility (NYMEX Oil 2006-09)

Figure 1: NYMEX Crude Oil daily price movements
Stochastic volatility (log-returns)
Stochastic volatility (log-returns)
A dynamic model

The standard dynamic model is the Exponentially Weighted Moving Average method of RiskMetrics: for a given half-life $0 < \lambda < 1$,

$$\sigma_t = \frac{\sum_{n=0}^{t-1} \lambda^n (r_{t-n} - \mu)^2}{\sum_{n=0}^{t-1} \lambda^n}$$

- Easy to program
- Mathematically inconsistent
The traditional GARCH(1,1) model is given by

\[ \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \]

The Glosten-Jagannathan-Runkle GARCH (GJR) is defined as

\[ \sigma_t^2 = \omega + (\alpha + \gamma I_{t-1}) \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \]

where \( I_t = 1 \) when \( r_t < \mu \), and 0 otherwise.
Model comparison

We compared three models and found that they score as follows:

1. GJR
2. Markov switching between two GED

\[\frac{\beta}{2\alpha \Gamma(1/\beta)} e^{-\left(|x-\mu|/\alpha\right)^\beta}\]

although both are close.
Correlations and Covariances

The interesting case is in the higher dimensional case

- To analyze correlations
- To model shifts from backwardation to contango

To go higher dimensional, there are three ways:

1. Multidimensional GARCH models
2. Multidimensional Markov Switching
3. Stochastic Principal Components
Consider the conditional covariance matrix of a random vector

\[ V_t = W \Sigma_t W^{-1} \]

where \( W \) is orthogonal and constant in time, and \( \Sigma_t \) is diagonal with diagonal entries \( \sigma_i \) that follow a GARCH model. The result is a parsimonious model

- Easy to work with
- Flexible
- Fits observed features, such as fat tails or autoregression
Stocks and Bonds

Stocks and Bonds

Cui Cui Amanda Luo (University of Toronto), Haofei Wang (R)

Stochastic Correlation in Energy Markets
Dynamic Principal Components

Consider the conditional covariance matrix of a random vector

$$V_t = W \Sigma_t W^{-1}$$

where $W$ is orthogonal and constant in time, and $\Sigma_t$ is diagonal with diagonal entries $\sigma_i$ that follow a CIR model

$$d_t \sigma_i = \alpha_i(\beta_i - \sigma_i) \, dt + \lambda \sqrt{\sigma_i} \, dW_i.$$ 