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Nassim Nicholas Taleb & Dhruv Madeka

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Author's Reply

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NASSIM NICHOLAS TALEB*†‡ and DHRUV MADEKA§

†Universa Investments, Miami, FL, USA
‡Tandon School of Engineering, New York University, New York, NY, USA
§Amazon, Inc., USA

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We are happy to respond to Clayton's letter, in spite of its confusions, as it will give us the opportunity to address more fundamental misunderstandings of the role of quantitative finance in general, and arbitrage pricing in particular, and proudly show how 'all roads lead to quantitative finance', that is, that arbitrage approaches are universal and applicable to all manner of binary forecasting. It also allows the second author to comment from his paper, Madeka (2017), which independently and simultaneously obtained similar results to Taleb (2018).

Incorrect claims

'*Taleb's criticism of popular forecast probabilities, specifically the election forecasts of FiveThirtyEight*⋯' and '*He [Taleb] claims this means the FiveThirtyEight forecasts must have "violate[d] arbitrage boundaries*"' are factually incorrect.

There is no mention of FiveThirtyEight in Taleb (2018), and Clayton must be confusing scientific papers with Twitter debates. The paper is an attempt at addressing elections in a rigorous manner, not journalistic discussion, and only mentions the 2016 election in one illustrative sentence.†

Let us, however, continue probing Clayton's other assertions, in spite of his confusion and the nature of the letter.

Incorrect arbitrage valuation

Clayton's claims either an error ('First, one of the "standard results" of quantitative finance that his election forecast assessments rely on is false', he initially writes), or, as he confusingly retracts, something 'only partially true'. Again, let us set aside that Taleb (2018) makes no 'assessment' of FiveThirtyEight's record and outline his reasoning.

Clayton considers three periods, $t_0 = 0$, an intermediate period t and a terminal one T, with $t_0 \leq t < T$. Clayton shows a special case of the distribution of the forward probability, seen at t_0, for time starting at $t = T/2$ and ending at T. It is a uniform distribution for that specific time period. In fact under his construction, using the probability integral transform, one can show that the probabilities follow what resembles a symmetric beta distribution with parameters a and b, and with $a = b$. When $t = T/2$, we have $a = b = 1$ (hence the uniform distribution). Before $T/2$ it has a $\cap$ shape, with Dirac at $t = t_0$. Beyond $T/2$ it has a $\cup$ shape, ending with two Dirac sticks at 0 and 1 (like a Bernoulli) when t is close to T (and close to an arcsine distribution with $a = b = \frac{1}{2}$ somewhere in between).

Clayton's construction is indeed misleading, since he analyzes the distribution of the price at time t with the filtration at time t_0, particularly when discussing arbitrage pricing and arbitrage pressures. Agents value options between t and T at time t (not period t_0), with an underlying price: under such constraint, the binary option automatically converges towards $\frac{1}{2}$ as $\sigma \to \infty$, and that for *any* value of the underlying price, no matter how far away from the strike price (or threshold). The σ here is never past realized, only future unrealized volatility. This can be seen within the framework presented in Taleb (2018) but also by taking any binary option pricing model. A price is not a probability (less even a probability distribution), but an expectation. Simply, as arbitrage operators, we look at *future* volatility given information about the underlying when pricing a binary option, not the distribution of probability itself in the unconditional abstract.

At infinite σ, it becomes all noise, and such a level of noise drowns all signals.

Another way to view the pull of uncertainty towards $\frac{1}{2}$ is in using information theory and the notion of maximum entropy under deep uncertainty: the entropy (I) of a

*Corresponding author. Email: nnt1@nyu.edu
† Incidentally, the problem with FiveThirtyEight is not changing probabilities from .55 to .85 within a 5-month period, it is performing abrupt changes within a much shorter timespan—and *that* was discussed in Madeka (2017).

Bernoulli distribution with probabilities p and $(1-p)$, $I = -((1-p)\log(1-p) + p\log(p))$ is maximal at $\frac{1}{2}$.†

To beat a $\frac{1}{2}$ pricing one needs to have enough information to beat the noise. As we will see in the next section, it is not easy.

Arbitrage counts

Another result from quantitative finance that puts bounds on the volatility of forecasting is as follows. Since election forecasts can be interpreted as a European binary option, we can exploit the fact that the price process of this option is bounded between 0 and 1 to make claims about the volatility of the price itself.

Essentially, if the price of the binary option varies too much, a simple trading strategy of buying low and selling high is guaranteed to produce a profit.‡. The argument can be summed up by noting that if we consider an arithmetic Brownian motion that's bounded between $[L, H]$:

$$dB_t = \sigma\, dW_t \tag{1}$$

The stochastic integral $2\int^T (B_0 - B_t)\, dB_t = \sigma^2 T - (B_T - B_0)^2$ can be replicated at zero cost, indicating that the value of B_T is bounded by the maximum value of the square difference on the right-hand side of the equation. That is, a forecaster who produces excessively volatile probabilities—if he or she is willing to trade on such a forecast (i.e. they have skin in the game)—can be arbitraged by following a strategy that sells (proportionally) when the forecast is too high and buys (proportionally) when the forecast is too low.

To conclude, any numerical probabilistic forecasting should be treated like a choice price—De Finetti's intuition is that forecasts should have skin in the game. Under these conditions, binary forecasting belongs to the rules of arbitrage and derivative pricing, well mapped in quantitative finance. Using a quantitative finance approach to produce binary forecasts does not prevent Bayesian methods (Taleb 2018 does not say probabilities should be $\frac{1}{2}$, only that there is a headwind towards that level owing to arbitrage pressures and constraints on how variable a forecast can be). It is just that there is one price that counts at the end, 1 or 0, which puts a structure on the updating.§

The reason Clayton might have trouble with quantitative finance could be that probabilities and underlying polls may not be martingales in real life; traded probabilities (hence real forecasts) must be martingales. Which is why in Taleb (2018) the process for the polls (which can be vague and nontradable) needs to be transformed into a process for probability in [0, 1].

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† Again, another way to view it: tradable probabilities are convex (to variables or parameters determining them) when they are close to 0, concave when they are close to 1. More uncertainty about the parameters or the variables pushes these probabilities away from 0 or 1 for arbitrage reasons.

‡ We take this result from Bruno Dupire's notes for his continuous time finance class at NYU's Courant Institute, particularly his final exam for the Spring of 2019.

§ Another way to see it, from outside our quantitative finance models: consider a standard probabilistic score. Let $X_1, \ldots, X_n$ be random variables $\in [0,1]$ and a B_T a constant $B_T \in \{0,1\}$, we have the λ score

$$\lambda_n = \frac{1}{n}\sum_{i=1}^{n}(x_i - B_T)^2,$$

which needs to be minimized (on a single outcome B_T). For any given B_T and an average forecast $\bar{x} = \sum_{i=1}^{n} x_i$, the minimum value of λ_n is reached for $x_1 = \cdots = x_n$. To beat a Dirac forecast $x_1 = \cdots = x_n = \frac{1}{2}$ for which $\lambda = \frac{1}{4}$ with a high variance strategy, one needs to have 75% accuracy. (Note that a uniform forecast has a score of $\frac{1}{3}$.) This gives us an intuition of the trade-off between volatility and signal.